Risk valuation for weather derivatives related to the energy market

Presentation Institut des Actuaires

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Outline

List of Papers

- Alfonsi, A., & Vadillo, N. (2024). A stochastic volatility model for the valuation of temperature derivatives. *IMA Journal of Management Mathematics*, dpae013.
- Alfonsi, A., & Vadillo, N. (2024). Risk valuation of quanto derivatives on temperature and electricity. *Applied Mathematical Finance*, 1–38.

Outline

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When winter comes what happens?

It is cold.

It is wet.

It is so snowy that the RER does not work.

So what do you do before it arrives?

Buy a warm coat

Ask for an umbrella for Christmas

Ensure a large stock of hot chocolate powder

What makes you think businesses do not cope with the same conditions?

Who?

- Energy producers and distributors
- Farmers and agro suppliers
- Construction
- Big industrials

Suffering from what?

- High demand of energy leading to price explosion
- Cold waves, frost and hail devastating crops
- Winter storms destroying sites
- Supply disruption

... and it is estimated that climate change could decrease up to 10% of total economic value worldwide by 2050 [26].

Introduction to weather derivatives

Insurance or derivative contracts defined by

- Annual Index I aggregate of...
- ...an underlying meteorological parameter: *e.g.* daily T
- and leads to a payoff according to a payoff function P

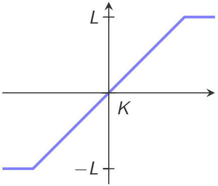

Figure: Swap
$P(I) = \max(-L, \min(L, \alpha * (I - K)))$

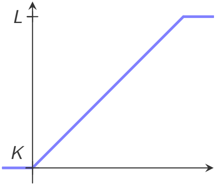

Figure: Call
$P(I) = \min(L, \alpha * (I - K)^{+})$

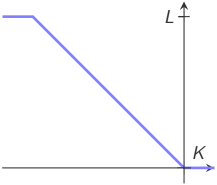

Figure: Put
$P(I) = \min(L, \alpha * (K - I)^{+})$

Figure: Payoff functions for three weather derivatives. Here I corresponds to the weather index, K to the strike, α to the notional or tick in (**$** per weather index unit) and L to the limit of payoff.

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Introduction to weather derivatives

Peril	Parameter	Weather index	Source
Heat wave, cold wave, frost	Temperature	HDD, CDD, CAT, minimum temperature, maximum temperature	Weather station, satellite data
Drought, excess of rainfall	Rainfall	Cumulative rainfall, number of rainy days	Weather station, satellite data
Lack of snow	Snowfall	Cumulative snowfall	Weather station
Lack of wind energy production	Windspeed	Wind power production	Weather station, satellite data
Lack of solar energy production	Solar radiation	Solar power production	Satellite data
Cyclone	Cyclone intensity or track	Cyclone intensity or track	Public agency
Earthquake	Earthquake intensity	Seismic intensity, magnitude	Public agency
Drought, wildfire	Vegetation indices	NDVI, soil moisture, burned index	Satellite data

Table: Summary of the different weather parameters and indices defining a weather derivative contract

Market characterisation

A recent market

1996 First over-the-counter (OTC) exchange

1999 CME launches open market

2006 A $45 billion worth market (95% temperature contracts)

2008 Slowdown and birth of quanto products

2020 Rebirth of interest and new actors

Characteristics of the current market

- CME unique open market with 72 contracts
- Considerably limited volumes with several days without transactions
- Mostly OTC exchanges (estimated 69% in 2004) with non-standardized products
- Involving specialized brokers and main reinsurers

$\Longrightarrow$ **No** market data.

Available data

Weather Data characteristic

- A diversity of meteorological parameters (temperature, rain, wind) and granularity (spacial and time).
- Both satellite and weather station data.
- Mainly provided by private companies who ensure data accessibility and quality check.

Data for our study

	Model (M)	Model (ETM)	Model (ETM)
Data	Average daily temperature	Average daily temperature	Day ahead prices
Time	01/01/1980 to 31/12/2020	5/01/2015 to 31/12/2018	5/01/2015 to 31/12/2018
Location	8 major European cities	Paris-Charles de Gaulle Milano-Linate	France North Italy
Source	Speedwell	Speedwell	ENTSO-E Gestore Mercati Energetici

How do we evaluate the risk related to weather derivatives?

Outline

What are temperature derivatives?

Contract structure

- The weather parameter is the average daily temperature T_t
- different indices related to average temperature

$$HDD := \sum_{t=t_1}^{t_2} \max(0, T_b - T_t), \; CAT := \sum_{t=t_1}^{t_2} T_t.$$

- and a deterministic payoff function f_T

$$f_T(HDD) := \min((HDD - HDD_{strike})^+, L) \tag{1}$$

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How to price temperature derivatives?

Outline

Our stochastic volatility model

Our stochastic volatility model

$$\begin{cases} T_t &= s(t) + \tilde{T}_t, \\ d\tilde{T}_t &= -\kappa \tilde{T}_t dt + \sqrt{\zeta_t}(\rho dW_t + \sqrt{1-\rho^2} dZ_t), \\ d\zeta_t &= -K(\zeta_t - \sigma^2(t))dt + \eta\sqrt{\zeta_t} dW_t, \end{cases} \tag{M}$$

where $(T_t)_{t\geq 0}$ temperature,
$(\tilde{T}_t)_{t\geq 0}$ deseasonalized and detrended temperature,
$(\zeta_t)_{t\geq 0}$ volatility,
W, Z are independent Brownian motions, $\kappa, \eta, K > 0$, $\rho \in [-1, 1]$,
s and σ^2 two deterministic functions in (3)

Our stochastic volatility model

Our stochastic volatility model

$$\begin{cases} T_t &= s(t) + \tilde{T}_t, \\ d\tilde{T}_t &= -\kappa \tilde{T}_t dt + \sqrt{\zeta_t}(\rho dW_t + \sqrt{1-\rho^2} dZ_t), \\ d\zeta_t &= -K(\zeta_t - \sigma^2(t))dt + \eta\sqrt{\zeta_t} dW_t, \end{cases} \tag{M}$$

Characteristics

- Integrates a trend and seasonal deterministic component s.
- Integrates an autoregressive component driven by κ.
- Integrates a volatility $(\zeta_t)_{t\geq 0}$ following a time-dependent Cox-Ingersoll-Ross (CIR) process [14].

Our stochastic volatility model

Our stochastic volatility model

$$\begin{cases} T_t &= s(t) + \tilde{T}_t, \\ d\tilde{T}_t &= -\kappa \tilde{T}_t dt + \sqrt{\zeta_t}(\rho dW_t + \sqrt{1-\rho^2} dZ_t), \\ d\zeta_t &= -K(\zeta_t - \sigma^2(t))dt + \eta\sqrt{\zeta_t} dW_t, \end{cases} \tag{M}$$

Advantages

- Brings larger flexibility on the volatility process.
- Time-continuous model $\Longrightarrow$ can be coupled with a model for energy or commodities.
- Affine model $\Longrightarrow$ efficient pricing methods based on Fourier techniques.
- Generalizes the Ornstein-Urhlenbeck model.

Daily temperature models

Ornstein–Uhlenbeck model [3]

$$\begin{cases} T_t & = s(t) + \tilde{T}_t, \\ \tilde{T}_t & = -\kappa \tilde{T}_t dt + \sigma(t) dW_t, \end{cases} \tag{2}$$

where $(T_t)_{t\geq 0}$ temperature,
$(\tilde{T}_t)_{t\geq 0}$ deseasonalized and detrended temperature,
W Brownian motion, $\kappa > 0$,
s and σ^2 two deterministic functions and $\xi_k = \frac{2\pi k}{365}$

$$\begin{cases} s(t) & = \alpha_0 + \beta_0 t + \sum_{k=1}^{K_s} \alpha_k \sin(\xi_k t) + \sum_{k=1}^{K_s} \beta_k \cos(\xi_k t), \\ \sigma^2(t) & = \gamma_0 + \sum_{k=1}^{K_{\sigma^2}} \gamma_k \sin(\xi_k t) + \sum_{k=1}^{K_{\sigma^2}} \delta_k \cos(\xi_k t) \end{cases} \tag{3}$$

Limits of the Ornstein-Uhlenbeck model

- Delete any potential long memory effects.
- Deviation from normal hypotheses: skewness, tail heaviness and volatility clustering pattern

Explored alternative models

Fractional Brownian motions [10]	$\longrightarrow$	$\hat{H} \approx 0.5$
Higher than one autoregressive terms	$\longrightarrow$	Autoregressive coefficients small & unstable
GARCH models	$\longrightarrow$	Same noise for temperature & volatility

Quantile quantile plot for model comparison

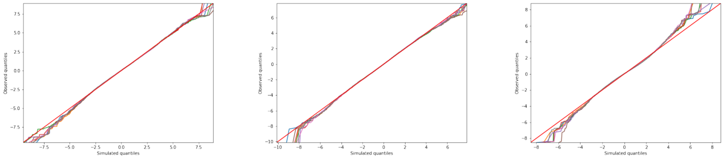

Figure: Quantile quantile plots for observed and 9 simulated noises for Stockholm, Paris and Rome for the Ornstein-Uhlenbeck model.

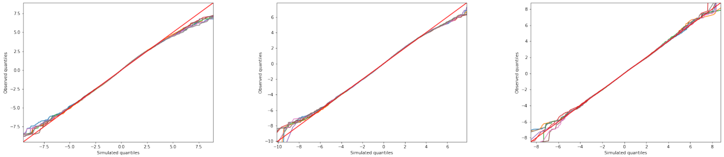

Figure: Quantile quantile plots for observed and 9 simulated noises for Stockholm, Paris and Rome for Model (M).

Outline

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Estimation challenges

Three major estimation challenges

Estimate the parameters of model (M)	$\longrightarrow$	CLSE
Compute the instantaneous volatility process $(\zeta_t)_{t\geq 0}$	$\longrightarrow$	ζ approximated by observed volatility $\hat{\zeta}_Q$
Evaluate robustness of the estimation	$\longrightarrow$	Robust estimation for T, less stable for ζ

Estimation Challenges

Challenge: Apply Conditional Least Squares Estimation (CLSE) to estimate parameters of Model (M) [9].

$$\sum_{i=0}^{N-1} \left(T_{(i+1)\Delta} - \mathbb{E}[T_{(i+1)\Delta} | T_{i\Delta}, \zeta_{i\Delta}]\right)^2 + \left(\zeta_{(i+1)\Delta} - \mathbb{E}[\zeta_{(i+1)\Delta} | \zeta_{i\Delta}]\right)^2$$

Remark

- Overbeck and Ryden [24] and Bolyog and Pap [9] proved convergence of the CLS estimators for the CIRs and Heston models
- Klimko and Nelson [21] show CLS estimator speed of convergence close to $O(N^{-1/2})$
- We show strong convergence of CLS estimators for the time-dependent CIR processes

Estimation Challenges

Challenge: Apply Conditional Least Squares Estimation (CLSE) to estimate parameters of Model (M) [9].
For s and κ,

$$\min_{\kappa,\alpha,\beta} \quad \sum_{i=0}^{N-1} \left(T_{(i+1)\Delta} - \mathbb{E}[T_{(i+1)\Delta} | T_{i\Delta}] \right)^2, \tag{4}$$

is given, if $\hat{\lambda}_2 \in (0,1)$, by

$$\begin{cases} \hat{\kappa} & = -\frac{1}{\Delta} \ln \hat{\lambda}_2 \\ \hat{\alpha}_0 & = \frac{\hat{\lambda}_0}{1-\hat{\lambda}_2} - \frac{\hat{\lambda}_1 \Delta}{(1-\hat{\lambda}_2)^2} \\ \hat{\beta}_0 & = \frac{\hat{\lambda}_1}{1-\hat{\lambda}_2} \\ \hat{\alpha}_1 & = \frac{\hat{\lambda}_3(\cos(\xi\Delta)-e^{-\hat{\kappa}\Delta})+\hat{\lambda}_4 \sin(\xi\Delta)}{(\cos(\xi\Delta)-e^{-\hat{\kappa}\Delta})^2+\sin^2(\xi\Delta)} \\ \hat{\beta}_1 & = \frac{\hat{\lambda}_4(\cos(\xi\Delta)-e^{-\hat{\kappa}\Delta})-\hat{\lambda}_3 \sin(\xi\Delta)}{(\cos(\xi\Delta)-e^{-\hat{\kappa}\Delta})^2+\sin^2(\xi\Delta)}, \end{cases}$$

where $X_{i\Delta} = (1, i\Delta, T_{i\Delta}, \sin(\xi i\Delta), \cos(\xi i\Delta))^T \in \mathbb{R}^5$ for $i \in \mathbb{N}$ and

$$\hat{\lambda} = \left(\sum_{i=0}^{N-1} X_{i\Delta} X_{i\Delta}^T \right)^{-1} \left(\sum_{i=0}^{N-1} X_{i\Delta} T_{(i+1)\Delta} \right).$$

Estimation challenges

Proof.

From Model (M), we have $\mathbb{E}[T_{t+\Delta}|\mathcal{F}_t] = T_t e^{-\kappa\Delta} + s(t+\Delta) - s(t)e^{-\kappa\Delta}$.
From the trigonometric identities we get:

$$s(t+\Delta) - e^{-\kappa\Delta}s(t) = \lambda_0 + \lambda_1 t + \lambda_3 \sin(\xi t) + \lambda_4 \cos(\xi t),$$

$$\begin{cases} \lambda_0 & = \alpha_0(1 - e^{-\kappa\Delta}) + \beta_0\Delta \\ \lambda_1 & = \beta_0(1 - e^{-\kappa\Delta}) \\ \lambda_2 & = e^{-\kappa\Delta} \\ \lambda_3 & = \alpha_1(\cos(\xi\Delta) - e^{-\kappa\Delta}) - \beta_1 \sin(\xi\Delta) \\ \lambda_4 & = \alpha_1 \sin(\xi\Delta) + \beta_1(\cos(\xi\Delta) - e^{-\kappa\Delta}), \end{cases}$$

With $X_{i\Delta} = (1, i\Delta, T_{i\Delta}, \sin(\xi i\Delta), \cos(\xi i\Delta))^T$ and λ_2 set to have $\mathbb{E}[T_{(i+1)\Delta}|\mathcal{F}_t] = \lambda^T X_{i\Delta}$. The minimization problem becomes

$$\min_{\lambda \in \mathbb{R}^5} \sum_{i=0}^{N-1} \left(T_{(i+1)\Delta} - \lambda^T X_{i\Delta}\right)^2.$$

□

Estimation challenges

Challenge: Compute the instantaneous volatility process $(\zeta_t)_{t\geq 0}$.

Work with realized volatilities $(\hat{\zeta}_{iQ\Delta})_{i\in[0,I]}$ [2]

$$\hat{\zeta}_{iQ\Delta} := \frac{1}{Q}\sum_{j=1}^{Q}\frac{2\hat{\kappa}}{1-e^{-2\hat{\kappa}\Delta}}\left(\tilde{T}_{(iQ+j)\Delta} - e^{-\hat{\kappa}\Delta}\,\tilde{T}_{(iQ+j-1)\Delta}\right)^2 \qquad (5)$$

$\hat{\zeta}_{iQ\Delta}$ realized volatility where $i \in \{0, \dots, \lfloor N/Q \rfloor - 1\}$

Remark

If ζ_t were frozen for $t \in [iQ\Delta, (i+1)Q\Delta]$, $\hat{\zeta}_{iQ\Delta}$ would be an unbiased estimator, i.e. $\mathbb{E}[\hat{\zeta}_{iQ\Delta}|\mathcal{F}_{iQ\Delta}] = \zeta_{iQ\Delta}$.

Estimation challenges

Challenge: Evaluate robustness of the estimation.

Analyse performance of the estimators applied to simulated time series:

$$\begin{cases} T_{(i+1)\Delta} = s((i+1)\Delta) + e^{-\kappa\Delta}(T_{i\Delta} - s(i\Delta)) + \sqrt{\dfrac{1-e^{-2\kappa\Delta}}{2\kappa}\dfrac{\zeta_{i\Delta}+\zeta_{(i+1)\Delta}}{2}} Z_i \\ \zeta_{(i+1)\Delta} = \phi(\zeta_{i\Delta}, \Delta, \sqrt{\Delta}Y_{i\Delta}). \end{cases}$$

- For T: discretize the integral of temperature dynamics
- For ζ: generalize Ninomiya-Victoir scheme for Cox-Ingersoll-Ross (CIR) processes [1]

$$\phi(\zeta_{i\Delta}, \Delta, \sqrt{\Delta}Y_{i\Delta}) = e^{-\frac{K\Delta}{2}}\left(\sqrt{(K\sigma^2((i+1/2)\Delta) - \frac{\eta^2}{4})\phi_K(\frac{\Delta}{2}) + \zeta_{i\Delta}e^{-\frac{K\Delta}{2}}} + \frac{\eta}{2}\sqrt{\Delta}Y_{i\Delta}\right)^2$$
$$+ (K\sigma^2((i+1/2)\Delta) - \frac{\eta^2}{4})\phi_K(\frac{\Delta}{2})$$

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Estimation challenges

Challenge: Evaluate robustness of the estimation.

For s and κ,

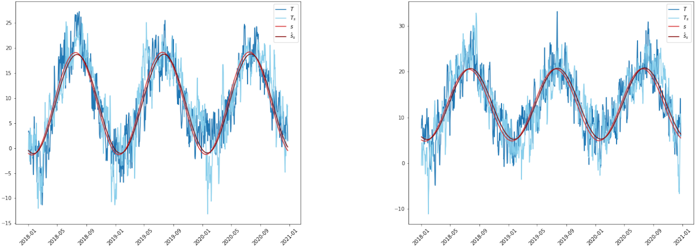

Figure: Estimation of the trend and seasonal function s on both real temperature T (blue) and simulated temperature T_S (light blue) for Stockholm (left) and Paris (right) and $Q = 10$.

City	Stockholm	Paris	Amsterdam	Berlin	Brussels	London	Rome	Madrid
κ	0.192	0.230	0.228	0.203	0.195	0.260	0.228	0.221
$\hat{\kappa}$	0.192	0.235	0.220	0.200	0.192	0.270	0.224	0.232

Table: Estimation of temperature autoregressive parameter from the simulated temperature path.

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Summary

- Model (M) answers to the limits of the Ornstein-Uhlenbeck model with a more conservative approach.
- CLSE enables to compute its parameters.
- Realized volatility ζ is approximated by an observed volatility $\hat{\zeta}$ which depends on time-window Q.
- Estimation is robust for parameters of s, κ and σ.
- Estimation of K and η^2 less stable. Chose a balanced $Q \Longrightarrow$ Need to understand sensitivity to these parameters.

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Outline

Pricing temperature derivatives

Prices under classic risk-neutral pricing theory correspond to:

$$\mathbb{E}_{\mathbb{Q}}\Big(D(t_0, t_2)\,\text{min}((HDD - HDD_{strike})^+, L)\Big)$$

where $\mathbb{Q}$ is the risk-neutral probability and $D(t_0, t_2)$ is a discount factor between t_0 and t_2.

However, temperature is not an asset traded on markets
$\Longrightarrow$ Risk neutral theory **cannot be applied**.
$\Longrightarrow$ We work on the **historical probability** world and analyse the payoff distribution.

Pricing temperature derivatives

Monte Carlo simulations

$$\begin{cases} T_{(i+1)\Delta} = s((i+1)\Delta) + e^{-\kappa\Delta}(T_{i\Delta} - s(i\Delta)) + \sqrt{\frac{1-e^{-2\kappa\Delta}}{2\kappa}\frac{\zeta_{i\Delta}+\zeta_{(i+1)\Delta}}{2}}Z_i \\ \zeta_{(i+1)\Delta} = \phi(\zeta_{i\Delta}, \Delta, \sqrt{\Delta}Y_i), \end{cases}$$

where $(Y_i, Z_i)_{i\geq 0} \sim \mathcal{N}(0, I)$ iid, $\psi_K(t) = \frac{1-e^{-Kt}}{K}$ and

$$\phi(\zeta_{i\Delta}, \Delta, \sqrt{\Delta}Y_i) = e^{-\frac{K\Delta}{2}}\left(\sqrt{\left(K\sigma^2((i+1/2)\Delta) - \frac{\eta^2}{4}\right)\psi_K\left(\frac{\Delta}{2}\right) + \zeta_{i\Delta}e^{-\frac{K\Delta}{2}}} + \frac{\eta}{2}\sqrt{\Delta}Y_i\right)^2 + \left(K\sigma^2((i+1/2)\Delta) - \frac{\eta^2}{4}\right)\psi_K\left(\frac{\Delta}{2}\right)$$

Fourier Transform Approach

- Computation of the characteristic function of Model (M).
- Fast Fourier Transform applied to Gil-Pelaez inversion formula [19].
- Combined with control variates method.

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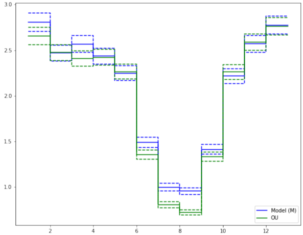
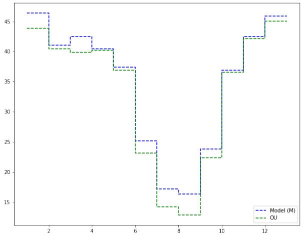
Monte-Carlo Approach



Mean with 95% confidence interval



Conditional Value at Risk at 95%

Figure: Different metrics of the payment distribution for 50, 000 Monte Carlo simulations, Paris, a cumulation period of a month, a forecast 30 days ahead and HDD_{strike} corresponding to a 90% quantile of the monthly HDD. Monte Carlo simulations are performed for both Model (M) and the Ornstein–Uhlenbeck model (2).

Fast Fourier Transform Approach

Characteristic Function [12]

Proposition

Let $0 \leq t \leq t'$. *Let* $(\tilde{T}, \zeta)$ *be the solution of* (M) *with* $\rho = 0$. *The characteristic function of* $(\tilde{T}_{t'}, \zeta_{t'})$ *given* $\mathcal{F}_t$ *is, for* $u_1, u_2, u_3 \in \mathbb{R}$,

$$\mathbb{E}\left[\exp\left(i[u_1 \tilde{T}_{t'} + u_2 \zeta_{t'} + u_3 \int_t^{t'} \tilde{T}_s ds]\right) | \mathcal{F}_t\right] = \exp(a_0(t, t') + a_1(t'-t)\tilde{T}_t + a_2(t'-t)\zeta_t), \quad (6)$$

where a_2 *is the unique solution on* $\mathbb{R}_+$ *of the time inhomogeneous autonomous Riccati equation*

$$a_2' = -K a_2 - \frac{1}{2}\left[u_1 \exp(-\kappa t) + u_3 \frac{1-\exp(-\kappa t)}{\kappa}\right]^2 + \frac{1}{2}\eta^2 a_2^2, \ \ a_2(0) = i u_2,$$

$a_1(t) = i u_1 \exp(-\kappa t) + i u_3 \frac{1-\exp(-\kappa t)}{\kappa}$ *and* $a_0(t, t') = K \int_t^{t'} \sigma^2(s) a_2(s) ds$.
Besides, the real part of $a_2(t)$ *remains nonpositive for all* $t \geq 0$.

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Fast Fourier Transform Approach

Characteristic Function [12]

Practically speaking, we freeze on each interval $[t_k, t_{k+1}]$ the value of the time inhomogenous term $t = \frac{t_k + t_{k+1}}{2}$. For $k < l$,

$$
\begin{cases}
a_2(t_{k+1}) = \Psi_k + \dfrac{2\sqrt{D_k}(\Psi_k - a_2(t_k))}{(\eta^2(\Psi_k - a_2(t_k)) - 2\sqrt{D_k})\exp(-\sqrt{D_k}\delta) - \eta^2(\Psi_k - a_2(t_k))} \\
a_1(t_k, t_l) = iu_1 \exp(-\kappa(t_l - t_k)) + iu_3 \dfrac{1 - \exp(-\kappa(t_l - t_k))}{\kappa} \\
a_0(t_k, t_l) \approx K \displaystyle\sum_{j=k}^{l-1} \frac{1}{2}[\sigma^2(t_j) a_2(t_l - t_j) + \sigma^2(t_{j+1}) a_2(t_l - t_{j+1})]\delta.
\end{cases}
\tag{6}
$$

where

$$
D_k = K^2 + \eta^2 \left(u_1 \exp\left(-\kappa \frac{t_k + t_{k+1}}{2}\right) + u_3 \frac{1 - \exp\left(-\kappa \frac{t_k + t_{k+1}}{2}\right)}{\kappa} \right)^2, \quad \Psi_k = \frac{K + \sqrt{D_k}}{\eta^2}.
$$

Fast Fourier Transform Approach

Control variates method for Monte-Carlo

Let consider

$$\underbrace{(HDD - HDD_{strike})^+}_{\text{Targeted estimator}} - \lambda \underbrace{((t_2 - t_1 + 1)T_b - HDD_{strike} - CAT)^+}_{\substack{\text{Control variable} \\ \text{Expectation easily computed thanks to} \\ \text{characteristic function (6) and FFT}}}$$

Optimal for

$$\lambda^* = \frac{Cov((HDD - HDD_{strike})^+, ((t_2 - t_1 + 1)T_b - HDD_{strike} - CAT)^+)}{Var(((t_2 - t_1 + 1)T_b - HDD_{strike} - CAT)^+)}.$$

Fast Fourier Transform Approach

Control variates method for Monte-Carlo

Month	1	2	3	4	5	6	7	8	9	10	11	12
Corr	1.00	1.00	1.00	1.00	0.94	0.66	0.38	0.33	0.66	0.97	1.00	1.00
VR	2.41e5	5.24e4	4.73e3	2.22e2	5.08	1.19	1.01	1.01	1.20	9.84	3.92e2	1.40e4

Table: Correlation and variance reduction (VR) brought by the control variates method for options computed during each month of 2019. Variance ratio corresponds to the variance of $(\sum_{t=t_1}^{t_2}(T_b - T_t)^+ - HDD_{strike})^+$ divided by the variance of the control variable.

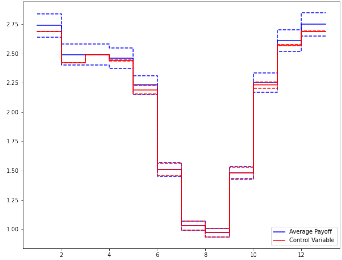

Figure: Expected payoffs forecasted 30 days ahead for a HDD derivative on each month of 2019 (blue) and for the control variable (red). We performed 50, 000 Monte Carlo simulations.

Extensions of the pricing approach

Comparison with **business pricing practices**: index modeling [20] [25]

- Coherence between the pricing methodologies.
- Daily modeling offers more precision and stability.

Perform parameter **sensitivity analysis**:

- Important sensitivity to κ. Price shrinkage when κ increases.
- Slight sensitivity to η^2 and K. Marginal impact in winter months.
- Sensitivity to $t_1 - t_0$. The more ahead we forecast the less information we have.
- Important sensitivity to moneyness of the product.

Summary

- Develop a stochastic volatility daily temperature model.
- Overpass estimation challenges thanks to CLSE and volatility approximation.
- Implement two pricing methodologies: Monte-Carlo and FFT.
- Boost Monte-Carlo simulations thanks to control variates.

Outline

What are quanto derivatives?

Contract structure

- A weather index corresponds to the aggregate of an underlying meteorological parameter T_t
- A price index corresponds to daily average electricity spot price $S_t = e^{X_t}$
- and a payoff depending on the product of two payoff functions f_S and f_T

$$Payoff := \sum_{t=t_1}^{t_2} f_S(S_t) \times f_T(T_t) \tag{7}$$

Main interest: hedge against both

- volumetric risk: wind [27], solar [5], temperature [11] [6] [15].
- price risk: natural gas [15] [6], electricity [4].

How to price quanto derivatives?

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Outline

A combined model for $(X_t, T_t)_{t\geq 0}$

A coupled model for electricity spot price $(X_t)_{t\geq 0}$ and average temperature $(T_t)_{t\geq 0}$

$$\begin{cases} d(X_t - \mu_X(t)) & = -\kappa_X(X_t - \mu_X(t)) + \lambda\sigma_T dW_t^T + dL_t^X \\ d(T_t - \mu_T(t)) & = -\kappa_T(T_t - \mu_T(t)) + \sigma_T dW_t^T \end{cases} \qquad \text{(ETM)}$$

where

- The deterministic function $\mu : \mathbb{R}_+ \to \mathbb{R}$ represents the trend and seasonality component,
- $\kappa > 0$ corresponds to the mean-reverting (or autoregressive) behaviour,
- W^T Brownian notion, L^X NIG Lévy noise, independent,
- $\lambda \in \mathbb{R}$ dependence parameter.

A combined model for $(X_t, T_t)_{t\geq 0}$

A coupled model for electricity spot price $(X_t)_{t\geq 0}$ and average temperature $(T_t)_{t\geq 0}$

$$\begin{cases} d(X_t - \mu_X(t)) & = -\kappa_X(X_t - \mu_X(t)) + \lambda\sigma_T dW_t^T + dL_t^X \\ d(T_t - \mu_T(t)) & = -\kappa_T(T_t - \mu_T(t)) + \sigma_T dW_t^T \end{cases} \qquad \text{(ETM)}$$

Advantages

- Convincing marginals
- Integrates dependence structure
- Maintains autoregressive mean-reverting time-continuous dynamics
  - $\Rightarrow$ Ease to estimate
  - $\Rightarrow$ Capacity to compute average payoff through FFT and explicit formulas

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Marginal model for log spot electricity price $(X_t)_{t\geq 0}$

A model inspired by Benth and Benth [7]

$$d(X_t - \mu_X(t)) = -\kappa_X(X_t - \mu_X(t)) + \lambda\sigma_T dW_t^T + dL_t^X \tag{8}$$

where

- The deterministic seasonality function μ_X

$$\mu_X(t) = \beta_0^X t + \alpha_1^X \sin(\xi t) + \beta_1^X \cos(\xi t) + \alpha_{DoW(t)}^{X,DoW}$$

  where $\xi = \frac{2\pi}{365}$ and $DoW(t) = \lfloor \frac{t}{\Delta} \rfloor \bmod p = 7$
- $\kappa_X > 0$ corresponds to the mean-reverting parameter,
- L^X is a Normal Inverse Gaussian distribution of parameters $(\alpha^X, \beta^X, \delta^X, m^X)$ centered.

Marginal model for log spot electricity price $(X_t)_{t\geq 0}$

First energy commodity models
First price energy commodity models are mean-reverting diffusion models from Schwartz [18]:

$$\begin{cases} X_t &= \mu(t) + \tilde{X}_t \\ \tilde{X}_t &= -\kappa \tilde{X}_t dt + \sigma dW_t \end{cases} \tag{9}$$

where $\mu(\cdot)$ deterministic function and W to a Brownian noise.

Model (9) is applied to electricity prices by Lucia and Schwartz [22]

Marginal model for log spot electricity price $(X_t)_{t \geq 0}$

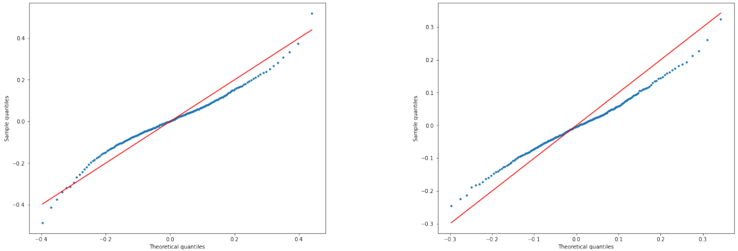

Figure: Quantile quantile plots for residuals compared with a theoretical quantiles of a normal distribution for French energy (left) and North Italian Energy (right).

Marginal model for log spot electricity price $(X_t)_{t \geq 0}$

Explored alternative models

Mean-reverting jump-diffusion models [16] [13] [17]	$\longrightarrow$	How to filter jumps?
Multi-factor mean-reverting models with Gaussian noise [22]	$\longrightarrow$	Still Brownian noises
Multi-factor mean-reverting models with Levy noise [23]	$\longrightarrow$	How to filter factors and then estimate convincingly?

Finally we keep a mean-reverting model with NIG noises as in Benth and Benth [7].

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Marginal model for log spot electricity price $(X_t)_{t \geq 0}$

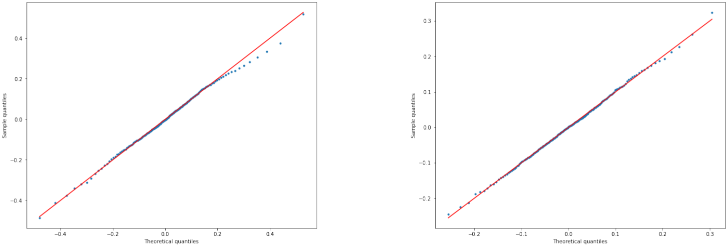

Figure: Quantile quantile plots for residuals compared with a theoretical quantiles of a normal inverse gaussian distribution for French energy (left) and North Italian Energy (right).

Marginal model for average temperature $(T_t)_{t\geq 0}$

A well established model developed by Benth et al. [8]

$$d(T_t - \mu_T(t)) = -\kappa_T(T_t - \mu_T(t)) + \sigma_T dW_t^T \qquad (10)$$

where

- The deterministic trend and seasonality function μ_T:

$$\mu_T(t) = \alpha_0^T + \beta_0^T t + \alpha_1^T \sin(\xi t) + \beta_1^T \cos(\xi t), \text{ where } \xi = \frac{2\pi}{365}.$$

- κ_T corresponds to the mean-reverting parameter,
- W^T is a Brownian motion and $\sigma_T > 0$ to the standard deviation of the noise.

Marginal model for average temperature $(T_t)_{t \geq 0}$

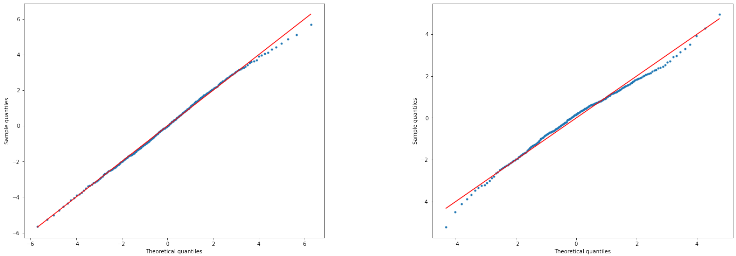

Figure: Quantile quantile plots for residuals compared with a theoretical quantiles of a normal distribution for Paris temperatures (left) and Milan temperatures (right).

Outline

Estimation procedure

Estimation in 5 steps

1. Deterministic terms κ and $\mu(\cdot)$ $\longrightarrow$ CLSE
2. Parameters of $(T_t)_{t\geq 0}$ $\longrightarrow$ MLE
3. Dependence parameter λ $\longrightarrow$ Observed covariance
4. NIG parameters of L^X $\longrightarrow$ CLSE on characteristic function
5. Goodness of fit of Model (ETM) $\longrightarrow$ χ^2-test goodness of fit

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1. Estimation of κ and $\mu(\cdot)$

Following Klimko and Nelson [21], we estimate κ and $\mu(\cdot)$ through Conditional Least Square (CLS)

$$\min_{\kappa,\alpha,\beta} \sum_{i=0}^{N-1} \left(X_{(i+1)\Delta} - \mathbb{E}[X_{(i+1)\Delta}|X_{i\Delta}]\right)^2, \tag{11}$$

is given, if $\hat{\lambda}_2 \in (0,1)$, by

$$\begin{cases} \hat{\kappa}_X &= -\ln \hat{\eta}_2 \\ \hat{\beta}_0^X &= \frac{\hat{\eta}_1}{1-\hat{\eta}_2} \\ \hat{\alpha}_1^X &= \frac{\hat{\eta}_3(\cos(\xi\Delta)-e^{-\hat{\kappa}_X\Delta})+\hat{\eta}_4\sin(\xi\Delta)}{(\cos(\xi\Delta)-e^{-\hat{\kappa}_X\Delta})^2+\sin^2(\xi\Delta)} \\ \hat{\beta}_1^X &= \frac{\hat{\eta}_4(\cos(\xi\Delta)-e^{-\hat{\kappa}_X\Delta})-\hat{\eta}_3\sin(\xi\Delta)}{(\cos(\xi\Delta)-e^{-\hat{\kappa}_X\Delta})^2+\sin^2(\xi\Delta)} \\ \hat{\alpha}_j^{X,DoW} &= \frac{1}{1-e^{-7\hat{\kappa}_X\Delta}}\sum_{k=0}^{6}(\hat{\eta}_{j+k}^{DoW} - \hat{\beta}_0)e^{-(6-k)\hat{\kappa}_X\Delta}, \end{cases}$$

where

$$\hat{\eta} = \left(\sum_{i=0}^{N-1} \Xi_{i\Delta}\Xi_{i\Delta}^\top\right)^{-1}\left(\sum_{i=0}^{N-1}\Xi_{i\Delta}X_{(i+1)\Delta}\right),$$

with $\Xi_{i\Delta} = (i\Delta, X_{i\Delta}, \sin(\xi i\Delta), \cos(\xi i\Delta), (1_{\{DoW(i\Delta)=j\}})_{0\le j\le 6}) \in \mathbb{R}^4 \times \{0,1\}^7$.

1. Estimation of κ and $\mu(\cdot)$

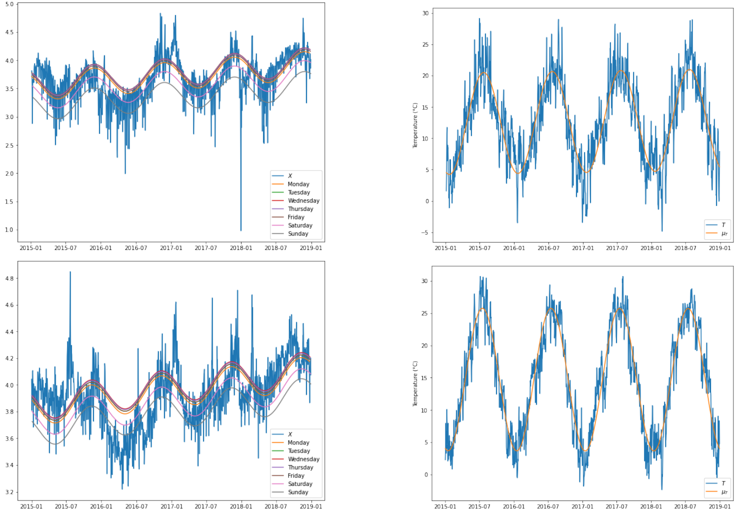


Figure: Fitted deterministic curve $\mu_X(\cdot)$ (left) and $\mu_T(\cdot)$ (right) for France (first row) and Italy (second row).

2. Estimation of the parameter of $(T_t)_{t\geq 0}$

Let consider the integral of dynamics $(T_t)_{t\geq 0}$ from Model (ETM), $\Delta > 0$ and $\tilde{T}_t = T_t - \mu_T(t)$:

$$\tilde{T}_{t+\Delta} = e^{-\kappa_T \Delta}\tilde{T}_t + \sigma_T \underbrace{\int_t^{t+\Delta} e^{-\kappa_T(t-u)} dW_u^T}_{\sim\mathcal{N}\left(m^T\sqrt{\frac{1-e^{-2\kappa_T\Delta}}{2\kappa_T}},\sigma_T^2\frac{1-e^{-2\kappa_T\Delta}}{2\kappa_T}\right)}$$

$\Longrightarrow$ Easy to estimate through Maximum Likelihood Estimation (MLE).

Results

Market	$\hat{m}^T$	$\hat{\sigma}_T^2$
France	10^{-15}	2.413
North Italy	10^{-16}	1.846

Table: Parameter estimation through the maximum likelihood estimation for dynamic of temperature normally distributed for Paris and Milan temperature.

3. Estimation of the dependence parameter of λ

Estimation of λ

From Model (ETM),

$$\hat{\lambda} = \frac{\hat{\kappa}_X + \hat{\kappa}_T}{\hat{\sigma}_T^2 (1 - e^{-(\hat{\kappa}_X + \hat{\kappa}_T)\Delta})} \widehat{Cov}$$

Results

Market	$\hat{\lambda}$
France	-0.007
North Italy	-0.002

Table: Estimated λ of Model (ETM) for France and North Italy.

4. Estimation of the parameter of $(X_t)_{t\geq 0}$

We apply CLS estimation applied to the (conditional) characteristic function:

$$\sum_{u}\sum_{t=0}^{N-1}\left|e^{iu(\tilde{X}_{t+\Delta}-e^{-\kappa_X\Delta}\tilde{X}_t)}-e^{-\frac{1}{2}\lambda^2\sigma_T^2\frac{1-e^{-2\kappa_X\Delta}}{2\kappa_X}u^2}\varphi(u;\Delta)\right|^2, \tag{12}$$

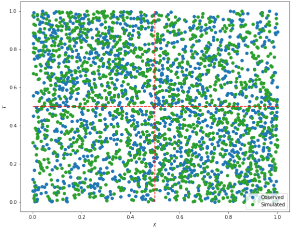
where φ the characteristic function of $\left(\int_t^{t+\Delta}e^{-\kappa_X(t+\Delta-v)}dL_v^X\right)$

$$\varphi(u;\Delta)=\exp\left(ium^X\frac{1-e^{-\kappa_X\Delta}}{\kappa_X}+\delta\gamma^X\Delta-\delta^X\int_t^{t+\Delta}\sqrt{(\alpha^X)^2-(\beta+iue^{-\kappa_X(t+\Delta-v)})^2}dv\right) \tag{13}$$

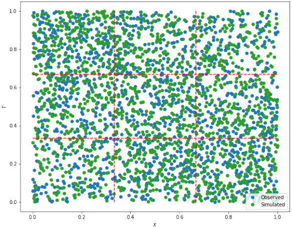
Remark

We compare CLS estimation through the characteristic function with alternative estimation through MLE and EM algorithm applied to an approximation of $\left(\int_t^{t+\Delta}e^{-\kappa_X(t+\Delta-v)}dL_v^X\right)$. All approaches show similar results.

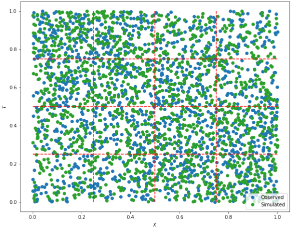
5. Goodness of fit



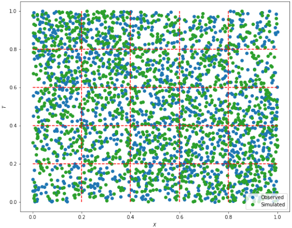
$p - value = 0.219$



$p - value = 0.892$



$p - value = 0.616$



$p - value = 0.124$

Figure: From top left to bottom right, χ^2 test performed on the distributions of real (blue) and simulated (green - based on $1,000,000$ simulations) ranked residuals for 4 and 25 categories for French data.

Summary

- CLS method is used to fit deterministic terms and NIG
- MLE is used to fit temperature
- Marginal models follow literature proposals and fit well 1D time-series
- Model (ETM) convincingly fits observations
- 9.43% and 4.34% of the standard deviation of the random term of the log energy spot price is explained by the temperature component for France and North Italy respectively

Outline

Characterisation of quanto derivatives

Following Cucu and al [15] and business practices, we define the quanto payoff:

$$Payoff := \sum_{t=t_1}^{t_2} f_S(S_t) \times f_T(T_t) \tag{14}$$

In particular, for

	Futures	Swaps	Single-sided options	Double-sided options
$f_S(S_t)$	S_t	$(S_t - \bar{S})$	S_t	$(S_t - \bar{S})^+$
$f_T(T_t)$	T_t	$(\bar{T} - T_t)$	$(\bar{T} - T_t)^+$	$(\bar{T} - T_t)^+$

Application: $\bar{T} = 18^\circ C$ (definition of HDD) and $\bar{S} = 50$ EUR/MWh.

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Pricing quanto derivatives

Again, we work under **historical probability** and analyse the payoff distribution.

Explicit or semi-explicit formulas deriving from the resolution of:

$$\mathbb{E}_{\mathbb{P}}\left(\sum_{t=t_1}^{t_2} f_S(S_t) \times f_T(T_t)\right)$$

Monte Carlo simulations

$$\begin{cases} X_{t+\Delta} &= \mu_X(t+\Delta) + e^{-\kappa_X \Delta}(X_t - \mu_X(t)) + \lambda\sigma_T\sqrt{\dfrac{1-e^{-2\kappa_X\Delta}}{2\kappa_X}}N_1 + e^{-\kappa_X\Delta/2}Z^X \\ T_{t+\Delta} &= \mu_T(t+\Delta) + e^{-\kappa_T \Delta}(T_t - \mu_T(t)) + \sigma_T\sqrt{\dfrac{1-e^{-2\kappa_T\Delta}}{2\kappa_T}}(\rho N_1 + \sqrt{1-\rho^2}N_2), \end{cases}$$

where $N_1 \sim \mathcal{N}(0,1)$, $N_2 \sim \mathcal{N}(0,1)$ and $Z^X \sim NIG(\alpha^X, \beta^X, \delta^X, -\frac{\delta^X\beta^X}{\gamma^X})$

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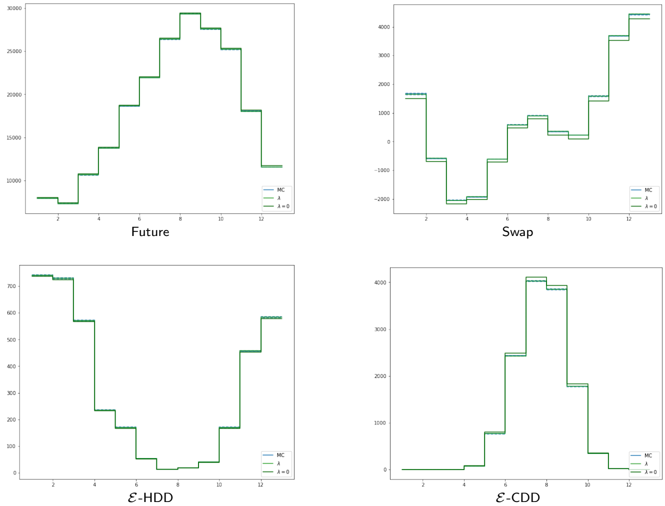

Figure: From top left to bottom right future, swap, $\mathcal{E}$-HDD and $\mathcal{E}$-CDD prices computed with 100,000 simulation-Monte Carlo (blue) and explicit formulas (green) methods. Each contract lasts one month of 2018. Time t_0 corresponds to 30 days ahead of the first day of the month, t_1 to the first day of the month and t_2 to the last day of the month.

Pricing quanto derivatives

For double sided quanto options we suggest a first order Taylor 's expansion on λ for $t \in [t_1, t_2]$

$$
\begin{aligned}
\mathcal{Q}(t_1, t_2) = \sum_{t=t_1}^{t_2} \Bigg(& \mathbb{E}_{\lambda=0}((S_t - \bar{S})^+ \mid \mathcal{F}_{t_0}) \times \Bigg(\Big(\bar{T} - \mu_T(t) - e^{-\kappa_T(t-t_0)}(T_{t_0} - \mu_T(t_0)) \Big) \times \\
& \Phi\Big(\frac{\bar{T} - \mu_T(t) - e^{-\kappa_T(t-t_0)}(T_{t_0} - \mu_T(t_0))}{\sigma_T k_T(t-t_0)} \Big) \\
& + \frac{\sigma_T k_T(t-t_0)}{\sqrt{2\pi}} \exp\Big(-\frac{1}{2}\Big(\frac{\bar{T} - \mu_T(t) - e^{-\kappa_T(t-t_0)}(T_{t_0} - \mu_T(t_0))}{\sigma_T k_T(t-t_0)} \Big)^2 \Big) \Bigg) \\
& - \Big(\mathbb{E}_{\lambda=0}((S_t - \bar{S})^+ \mid \mathcal{F}_{t_0}) + \bar{S}\mathbb{P}_{\lambda=0}\left(S_t \geq \bar{S} \mid \mathcal{F}_{t_0} \right) \Big) \times \\
& \sigma_T^2 k_{XT}(t-t_0)^2 \Phi\Big(\frac{\bar{T} - \mu_T(t) - e^{-\kappa_T(t-t_0)}(T_{t_0} - \mu_T(t_0))}{-\sigma_T k_T(t-t_0)} \Big) \Big) \lambda \Bigg) + o(\lambda)
\end{aligned}
\tag{15}
$$

where Φ is the cumulative distribution function of the standard Gaussian distribution, $k_T(\cdot)$, $k_X(\cdot)$ and $k_{XT}(\cdot)$ are defined in function of κ_X and κ_T.

Pricing quanto derivatives

For double sided quanto options we suggest a first order Taylor 's expansion on λ for $t \in [t_1, t_2]$

$$
\begin{aligned}
\mathcal{Q}(t_1, t_2) = \sum_{t=t_1}^{t_2} \Bigg(& \mathbb{E}_{\lambda=0}((S_t - \bar{S})^+ \mid \mathcal{F}_{t_0}) \times \Bigg(\Big(\bar{T} - \mu_T(t) - e^{-\kappa_T (t-t_0)}(T_{t_0} - \mu_T(t_0)) \Big) \times \\
& \Phi\Big(\frac{\bar{T} - \mu_T(t) - e^{-\kappa_T (t-t_0)}(T_{t_0} - \mu_T(t_0))}{\sigma_T k_T(t-t_0)} \Big) \\
& + \frac{\sigma_T k_T(t-t_0)}{\sqrt{2\pi}} \exp\Big(-\frac{1}{2}\Big(\frac{\bar{T} - \mu_T(t) - e^{-\kappa_T (t-t_0)}(T_{t_0} - \mu_T(t_0))}{\sigma_T k_T(t-t_0)} \Big)^2 \Big) \Bigg) \\
& - \Big(\mathbb{E}_{\lambda=0}((S_t - \bar{S})^+ \mid \mathcal{F}_{t_0}) + \bar{S}\mathbb{P}_{\lambda=0}\left(S_t \geq \bar{S} \mid \mathcal{F}_{t_0}\right) \Big) \times \\
& \sigma_T^2 k_{XT}(t-t_0)^2 \Phi\Big(\frac{\bar{T} - \mu_T(t) - e^{-\kappa_T (t-t_0)}(T_{t_0} - \mu_T(t_0))}{-\sigma_T k_T(t-t_0)} \Big) \Big) \lambda \Bigg) + o(\lambda)
\end{aligned}
\tag{15}
$$

where $\mathbb{E}_{\lambda=0}((S_t - \bar{S})^+ \mid \mathcal{F}_{t_0})$ is computed through Carr Madan formula [12, Equations (5) and (6)] and $\mathbb{P}_{\lambda=0}\left(S_t \geq \bar{S} \mid \mathcal{F}_{t_0}\right)$ through Gil-Pelaez [19] inversion formula.

Pricing quanto derivatives

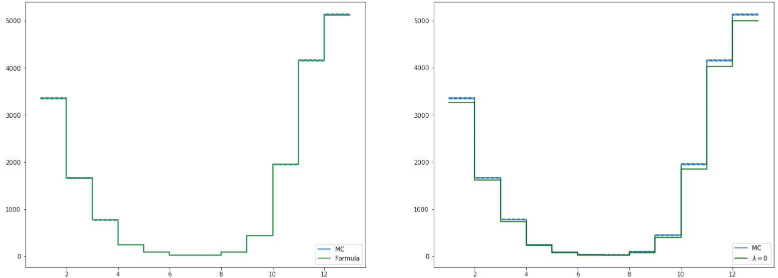

Figure: Quanto prices computed with 100,000 simulation-Monte Carlo (blue) and Equation (15) (green) methods. On the left the price corresponds to Formula (15). On the right only the first term of the Taylor development in Equation (15) is considered. This is equivalent to consider $\lambda = 0$. Each contract lasts a month of 2018. Time t_0 corresponds to 30 days ahead of the first day of the month, t_1 to the first day of the month and t_2 to the last day of the month. The computation of the derivatives through the formulas is around 6 times faster than using Monte Carlo simulations.

Outline

Static hedging of quanto derivatives

Daily double-sided quantos
Under Model (ETM) and for $\Delta > 0$, let consider the portfolio

$$\underbrace{(S_{t+\Delta} - \bar{S})^+ (\bar{T} - T_{t+\Delta})^+}_{\text{double-sided option quanto}} - \underbrace{d^0_{t,t+\Delta}}_{\text{cash}} - d^1_{t,t+\Delta} \underbrace{(\bar{T} - T_{t+\Delta})^+}_{\text{HDD future}} - d^2_{t,t+\Delta} \underbrace{(S_{t+\Delta} - \bar{S})^+}_{\text{call on spot}}$$

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Static hedging of quanto derivatives

Daily double-sided quantos

Under Model (ETM) and for $\Delta > 0$, let consider the portfolio

$$\mathbb{E}\Big[\Big((S_{t+\Delta}-\bar{S})^{+}(\bar{T}-T_{t+\Delta})^{+}-d^{0}_{t,t+\Delta}-d^{1}_{t,t+\Delta}(\bar{T}-T_{t+\Delta})^{+}-d^{2}_{t,t+\Delta}(S_{t+\Delta}-\bar{S})^{+}\Big)^{2}|\mathcal{F}_t\Big].$$

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Static hedging of quanto derivatives

Daily double-sided quantos
Under Model (ETM) and for $\Delta > 0$, let consider the portfolio

$$\mathbb{E}\Big[\Big((S_{t+\Delta} - \bar{S})^+(\bar{T} - T_{t+\Delta})^+ - d^0_{t,t+\Delta} - d^1_{t,t+\Delta}(\bar{T} - T_{t+\Delta})^+ - d^2_{t,t+\Delta}(S_{t+\Delta} - \bar{S})^+\Big)^2 | \mathcal{F}_t\Big].$$

$(d^0_{t,t+\Delta}, d^1_{t,t+\Delta}, d^2_{t,t+\Delta})$ minimising the above quadratic criterion is the unique solution of the linear system below:

$$\begin{bmatrix} 1 & \mathbb{E}[(\bar{T} - T_{t+\Delta})^+ | \mathcal{F}_t] & \mathbb{E}[(S_{t+\Delta} - \bar{S})^+ | \mathcal{F}_t] \\ \mathbb{E}[(\bar{T} - T_{t+\Delta})^+ | \mathcal{F}_t] & \mathbb{E}[((\bar{T} - T_{t+\Delta})^+)^2 | \mathcal{F}_t] & \mathbb{E}[(S_{t+\Delta} - \bar{S})^+(\bar{T} - T_{t+\Delta})^+ | \mathcal{F}_t] \\ \mathbb{E}[(S_{t+\Delta} - \bar{S})^+ | \mathcal{F}_t] & \mathbb{E}[(S_{t+\Delta} - \bar{S})^+(\bar{T} - T_{t+\Delta})^+ | \mathcal{F}_t] & \mathbb{E}[((S_{t+\Delta} - \bar{S})^+)^2 | \mathcal{F}_t] \end{bmatrix} \begin{bmatrix} d^0_{t,t+\Delta} \\ d^1_{t,t+\Delta} \\ d^2_{t,t+\Delta} \end{bmatrix} = \begin{bmatrix} \mathbb{E}[(S_{t+\Delta} - \bar{S})^+(\bar{T} - T_{t+\Delta})^+ | \mathcal{F}_t] \\ \mathbb{E}[(S_{t+\Delta} - \bar{S})^+((\bar{T} - T_{t+\Delta})^+)^2 | \mathcal{F}_t] \\ \mathbb{E}[((S_{t+\Delta} - \bar{S})^+)^2(\bar{T} - T_{t+\Delta})^+ | \mathcal{F}_t] \end{bmatrix}$$

Static hedging of quanto derivatives

Daily double-sided quantos

Under Model (ETM) and for $\Delta > 0$, let consider the portfolio

$$\mathbb{E}\Big[\Big((S_{t+\Delta} - \bar{S})^+(\bar{T} - T_{t+\Delta})^+ - d^0_{t,t+\Delta} - d^1_{t,t+\Delta}(\bar{T} - T_{t+\Delta})^+ - d^2_{t,t+\Delta}(S_{t+\Delta} - \bar{S})^+\Big)^2 |\mathcal{F}_t\Big].$$

$(d^0_{t,t+\Delta}, d^1_{t,t+\Delta}, d^2_{t,t+\Delta})$ minimising the above quadratic criterion is the unique solution of the linear system below:

$$\begin{bmatrix} 1 & \mathbb{E}[(\bar{T} - T_{t+\Delta})^+|\mathcal{F}_t] & \mathbb{E}[(S_{t+\Delta} - \bar{S})^+|\mathcal{F}_t] \\ \mathbb{E}[(\bar{T} - T_{t+\Delta})^+|\mathcal{F}_t] & \mathbb{E}[((\bar{T} - T_{t+\Delta})^+)^2|\mathcal{F}_t] & \mathbb{E}[(S_{t+\Delta} - \bar{S})^+(\bar{T} - T_{t+\Delta})^+|\mathcal{F}_t] \\ \mathbb{E}[(S_{t+\Delta} - \bar{S})^+|\mathcal{F}_t] & \mathbb{E}[(S_{t+\Delta} - \bar{S})^+(\bar{T} - T_{t+\Delta})^+|\mathcal{F}_t] & \mathbb{E}[((S_{t+\Delta} - \bar{S})^+)^2|\mathcal{F}_t] \end{bmatrix} \begin{bmatrix} d^0_{t,t+\Delta} \\ d^1_{t,t+\Delta} \\ d^2_{t,t+\Delta} \end{bmatrix} = \begin{bmatrix} \mathbb{E}[(S_{t+\Delta} - \bar{S})^+(\bar{T} - T_{t+\Delta})^+|\mathcal{F}_t] \\ \mathbb{E}[(S_{t+\Delta} - \bar{S})^+((\bar{T} - T_{t+\Delta})^+)^2|\mathcal{F}_t] \\ \mathbb{E}[((S_{t+\Delta} - \bar{S})^+)^2(\bar{T} - T_{t+\Delta})^+|\mathcal{F}_t] \end{bmatrix}$$

Computed through first order Taylor expansion in λ, Carr Madan formula [12] and Gil-Pelaez [19] inversion formula.

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Static hedging of quanto derivatives

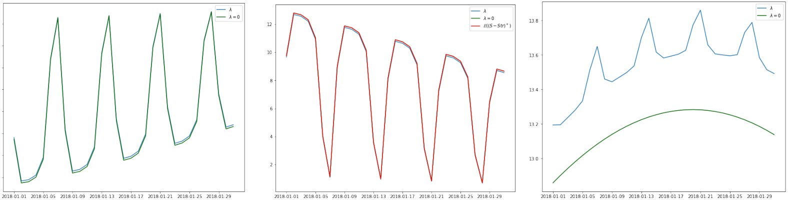

Figure: From top left to bottom right, $d^0_{t_0,t_1+i\Delta}$, $d^1_{t_0,t_1+i\Delta}$ and $d^2_{t_0,t_1+i\Delta}$ starting from 1st January 2018 (t_1), with $t_0 = t_1 - 30$ and with $t_0 = t_1 - 30$, $\Delta = 1$ and $i = 0, \ldots, 30$.

Static hedging of quanto derivatives

Monthly double-sided quantos

Let know consider the monthly portfolio:

$$\sum_{i=1}^{31}(S_{t_1+i\Delta}-\bar{S})^+(\bar{T}-T_{t_1+i\Delta})^+ - d^0_{t_0,t_1+(i-1)\Delta} - d^1_{t_0,t_1+(i-1)\Delta}(\bar{T}-T_{t_1+i\Delta})^+ - d^2_{t_0,t_1+(i-1)\Delta}(S_{t_1+i\Delta}-\bar{S})^+$$

and perform daily hedging as above to get:

	Without hedging	Hedging	Hedging $\lambda = 0$
January	$-3,358$	0.208	-93.072
May	-89.174	-0.113	-12.283

	Without hedging	Hedging	Hedging $\lambda = 0$
January	2, 197	391	394
May	177	98	100

Table: Average (left) and standard deviation (right) of $\sum_{i=1}^{31} d^0_{t_0,t_1+(i-1)\Delta} + d^1_{t_0,t_1+(i-1)\Delta}(\bar{T}-T_{t_1+i\Delta})^+ + d^2_{t_0,t_1+(i-1)\Delta}(S_{t_1+i\Delta}-\bar{S})^+ - (S_{t_1+i\Delta}-\bar{S})^+(\bar{T}-T_{t_1+i\Delta})^+$ for portfolio optimisation starting on 1st January 2018 (for t_1 on the left) and 1st May 2018 (for t_1 on the right), with $t_0 = t_1 - 30$ and lasting the whole month.

Static hedging of quanto derivatives

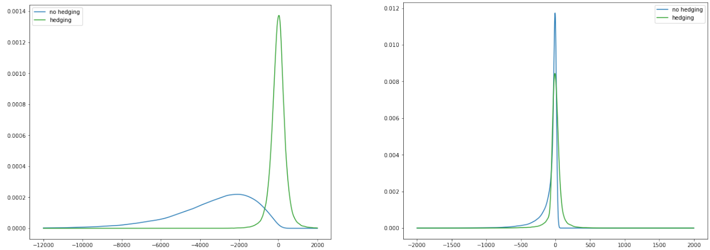

Figure: Empirical density of $\sum_{i=1}^{31} -(S_{t_1+i\Delta} - \bar{S})^+(\bar{T} - T_{t_1+i\Delta})^+$ (blue) and $\sum_{i=1}^{31} d^0_{t_0,t_1+(i-1)\Delta} + d^1_{t_0,t_1+(i-1)\Delta}(\bar{T} - T_{t_1+i\Delta})^+ + d^2_{t_0,t_1+(i-1)\Delta}(S_{t_1+i\Delta} - \bar{S})^+ - (S_{t_1+i\Delta} - \bar{S})^+(\bar{T} - T_{t_1+i\Delta})^+$ (green) for portfolio optimisation starting on 1st January 2018 (for t_1 on the left) and 1st May 2018 (for t_1 on the right), with $t_0 = t_1 - 30$ and lasting the whole month.

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Summary

- Develop a coupled model for daily average temperature and electricity price.
- Overpass estimation challenges thanks to MLE and CLSE on characteristic function.
- Obtain explicit and semi-explicit formulas for futures, swap, single-sided and double-sided options.
- Show risk hedging capacity of single-sided ($\mathcal{E}$-HDD) and double-sided quanto options.

Outline

Summary

Main contributions

- We study risk valuation for two weather derivatives: classic temperature derivatives and quanto derivatives combining temperature and electricity price.
- For both, we propose efficient models to describe the dynamics of the underlyings leveraging and extending literature proposals.
- We address succesfully estimation, pricing and hedging challenges.

More broadly

- We contribute to the establishment of a mathematical framework to better understand risk related to weather derivatives.
- We suggest direct applications of these models to practitioners.

Thank you

References I

A. Alfonsi.
High order discretization schemes for the CIR process: application to affine term structure and heston models.
Mathematics of Computation, 79(269):209–237, 2010.

R. Azencott, P. Ren, and I. Timofeyev.
Realised volatility and parametric estimation of Heston SDEs.
Finance and Stochastics, 24:723–755, 2020.

F. E. Benth and J. Benth, Šaltytė-Benth.
The volatility of temperature and pricing of weather derivatives.
Quantitative Finance, 7(5):553–561, 2007.

F. E. Benth, G. Deelstra, and A. S. Kozpınar.
Pricing energy quanto options in the framework of Markov-modulated additive processes.
IMA Journal of Management Mathematics, 34(1):187–220, 2023.

References II

F. E. Benth and N. Ibrahim.
Stochastic modeling of photovoltaic power generation and electricity prices.
Journal of Energy Markets, Forthcoming, 2017.

F. E. Benth, N. Lange, and T. A. Myklebust.
Pricing and hedging quanto options in energy markets.
Journal of Energy Markets, 8(1), 2015.

F. E. Benth and J. Šaltytė-Benth.
The normal inverse gaussian distribution and spot price modelling in energy markets.
International journal of theoretical and applied finance, 7(02):177–192, 2004.

F. E. Benth, J. Šaltytė-Benth, and S. Koekebakker.
Putting a price on temperature.
Scandinavian Journal of Statistics, 34(4):746–767, 2007.

References III

B. Bolyog and G. Pap.
On conditional least squares estimation for affine diffusions based on continuous time observations.
Statistical Inference for Stochastic Processes, 22(1):41–75, 2019.

D. C. Brody, J. Syroka, and M. Zervos.
Dynamical pricing of weather derivatives.
Quantitative finance, 2(3):189, 2002.

M. Caporin, J. Preś, and H. Torro.
Model based monte carlo pricing of energy and temperature quanto options.
Energy Economics, 34(5):1700–1712, 2012.

P. Carr and D. Madan.
Option valuation using the fast Fourier transform.
Journal of computational finance, 2(4):61–73, 1999.

References IV

A. Cartea and M. G. Figueroa.
Pricing in electricity markets: a mean reverting jump diffusion model with seasonality.
Applied Mathematical Finance, 12(4):313–335, 2005.

J. C. Cox, J. E. Ingersoll Jr, and S. A. Ross.
An intertemporal general equilibrium model of asset prices.
Econometrica: Journal of the Econometric Society, pages 363–384, 1985.

L. Cucu, R. Döttling, P. Heider, and S. Maina.
Managing temperature-driven volume risks.
Journal of Energy Markets, 9(2), 2016.

S. Deng.
Stochastic models of energy commodity prices and their applications: Mean-reversion with jumps and spikes.
University of California Energy Institute Berkeley, 2000.

References V

- H. Geman and A. Roncoroni.
  Understanding the fine structure of electricity prices.
  The Journal of Business, 79(3):1225–1261, 2006.
- R. Gibson and E. S. Schwartz.
  Stochastic convenience yield and the pricing of oil contingent claims.
  The Journal of Finance, 45(3):959–976, 1990.
- J. Gil-Pelaez.
  Note on the inversion theorem.
  Biometrika, 38(3-4):481–482, 1951.
- S. Jewson and A. Brix.
  Weather derivative valuation: the meteorological, statistical, financial and mathematical foundations.
  Cambridge University Press, 2005.
- L. A. Klimko and P. I. Nelson.
  On conditional least squares estimation for stochastic processes.
  The Annals of statistics, pages 629–642, 1978.

References VI

J. J. Lucia and E. S. Schwartz.
Electricity prices and power derivatives: Evidence from the nordic power exchange.
Review of derivatives research, 5(1):5–50, 2002.

T. Meyer-Brandis and P. Tankov.
Multi-factor jump-diffusion models of electricity prices.
International Journal of Theoretical and Applied Finance, 11(05):503–528, 2008.

L. Overbeck and T. Ryden.
Estimation in the Cox-Ingersoll-Ross model.
Econometric Theory, 13(3):430–461, 1997.

F. Schiller, G. Seidler, and M. Wimmer.
Temperature models for pricing weather derivatives.
Quantitative Finance, 12(3):489–500, 2012.

Swiss Re Institute.
The economics of climate change, April 2021.

References VII

Y. Yamada and T. Matsumoto.
Construction of mixed derivatives strategy for wind power producers.
Energies, 16(9):3809, 2023.

Estimation Challenges

2. **Estimate σ^2 and K**

$$\min_{K,\gamma,\delta} \quad \sum_{i=0}^{N-1} \left(\zeta_{(i+1)\Delta} - \mathbb{E}[\zeta_{(i+1)\Delta}|\zeta_{i\Delta}]\right)^2 \tag{16}$$

3. **Estimate η^2**

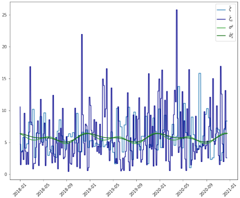
$$\min_{\eta^2} \sum_{i=0}^{N-1} \Big((\zeta_{(i+1)\Delta} - \mathbb{E}[\zeta_{(i+1)\Delta}|\zeta_{i\Delta}])^2 - \mathbb{E}[\left(\zeta_{(i+1)\Delta} - \mathbb{E}[\zeta_{(i+1)\Delta}|\zeta_{i\Delta}]\right)^2 |\zeta_{i\Delta}] \Big)^2 \tag{17}$$

4. **Estimate ρ**

$$\begin{aligned} \min_{\rho} \sum_{i=0}^{N-1} \Big(& (T_{(i+1)\Delta} - \mathbb{E}[T_{(i+1)\Delta}|\mathcal{F}_{i\Delta}])(\zeta_{(i+1)\Delta} - \mathbb{E}[\zeta_{(i+1)\Delta}|\mathcal{F}_{i\Delta}]) \\ & - \mathbb{E}[(T_{(i+1)\Delta} - \mathbb{E}[T_{(i+1)\Delta}|\mathcal{F}_{i\Delta}])(\zeta_{(i+1)\Delta} - \mathbb{E}[\zeta_{(i+1)\Delta}|\zeta_{i\Delta}]|\mathcal{F}_{i\Delta}]) \Big)^2 \end{aligned} \tag{18}$$

Estimation Challenges

Robustness of estimations of σ^2 independently of Q



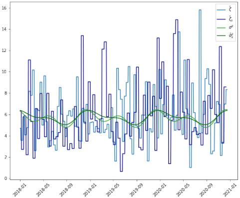


Figure: Plots of observed volatility process $\hat{\zeta}$ (blue) and simulated volatility processes $\hat{\zeta}_s$ (dark blue) for Paris for averaging windows Q equals 5 (left) and 12 (right).

Sensitivity of $\hat{K}$ and $\hat{\eta}^2$ to Q

City	$\hat{K}$	$\hat{K}_{Q=1}$	$\hat{K}_{Q=2}$	$\hat{K}_{Q=5}$	$\hat{K}_{Q=8}$	$\hat{K}_{Q=10}$	$\hat{K}_{Q=12}$
Stockholm	0.147	2.261	0.886	0.301	0.190	0.157	0.140
Paris	0.396	2.853	1.336	0.552	0.403	0.286	0.265

City	$\hat{\eta}^2$	$\hat{\eta}^2_{Q=1}$	$\hat{\eta}^2_{Q=2}$	$\hat{\eta}^2_{Q=5}$	$\hat{\eta}^2_{Q=8}$	$\hat{\eta}^2_{Q=10}$	$\hat{\eta}^2_{Q=12}$
Stockholm	0.629	56.229	12.288	2.123	0.896	0.644	0.499
Paris	1.043	56.429	13.385	2.506	1.156	0.690	0.531

Table: Estimation of K and η^2 for different averaging time windows Q.

Fast Fourier Transform Approach

Characteristic Function [12]

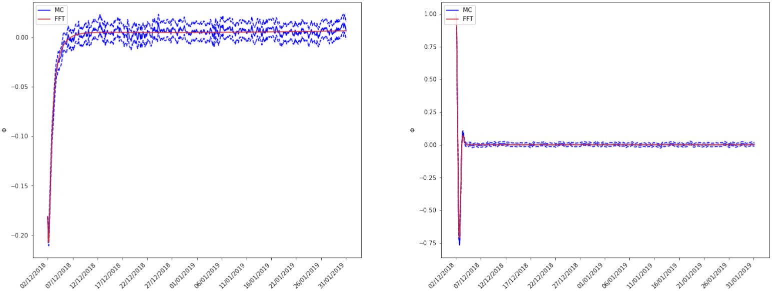


Figure: Characteristic function $\mathbb{E}\left[\exp\left(iu_1 \tilde{T}_{t'}\right)\right]$ (left) and $\mathbb{E}\left[\exp\left(iu_3 \int_t^{t'} \tilde{T}_s \, ds\right) | \mathcal{F}_t\right]$ (right) for Paris temperature during January 2019 for an observation time 30 days ahead and $\delta = 0.1$ day.

Fast Fourier Transform Approach

Fast Fourier Transform for pricing HDD and CAT options
Let note that during winter, we mostly have $T_t \leq T_b$. Hence,

$$\mathbb{E}[(HDD - HDD_{strike})^+] \approx \mathbb{E}[((t_2 - t_1 + 1)T_b - HDD_{strike} - CAT)^+]. \quad (19)$$

Let first compute the characteristic function of CAT

$$\begin{aligned}\Phi(u) &\approx e^{iu\sum_{t=t_1}^{t_2} s(t)} \mathbb{E}[e^{iu\int_{t_1}^{t_2+1} \tilde{T}_t dt} | \mathcal{F}_{t_0}] \\ &\approx e^{iu\sum_{t=t_1}^{t_2} s(t)} \exp(a_0(t_1, t_2+1)) \exp(\check{a}_0(t_0, t_1) + \check{a}_1(t_1 - t_0)\tilde{T}_{t_0} + \check{a}_2(t_1 - t_0)\zeta_{t_0})\end{aligned}$$

where

- a_0 is deduced by applying Proposition 32 with $u_1 = u_2 = 0$ and $u_3 = u$
- $\check{a}_0, \check{a}_1, \check{a}_2$ by applying Proposition 32 with $u_1 = u\frac{1 - e^{-\kappa(t_2+1-t_1)}}{\kappa}$, $u_2 = -ia_2(t_2 + 1 - t_1)$ and $u_3 = 0$

Fast Fourier Transform Approach

Fast Fourier Transform for pricing HDD and CAT options
Finally, we can use

$$\mathbb{E}[((t_2 - t_1 + 1)T_b - HDD_{strike} - CAT)^+] \approx \delta_x \left(\sum_{k=0}^{N-2} \mathbb{P}(\tilde{T}_t \leq x_k) + \frac{1}{2}\mathbb{P}(\tilde{T}_t \leq x_{N-1}) \right). \quad (20)$$

which can be computed through FFT inverse

$$\mathbb{P}(\tilde{T} \leq x_k) \approx \frac{1}{2} - \frac{\delta_v}{\pi}\Re\left(\sum_{j=0}^{N-1} \frac{e^{-iv_{j+1/2}x_k}\Phi(v_{j+1/2})}{iv_{j+1/2}} \right)$$

where we define $\delta_x \delta_v = \frac{2\pi}{N}, v_{j+1/2} = (j+1/2)\delta_v,\ x_k = (t_2 - t_1 + 1)T_b - HDD_{strike} + (k - N + 1)\delta_x, j \in \{0, N-1\}, k \in \{0, \ldots, N-1\}$

Fast Fourier Transform Approach

Fast Fourier Transform for pricing HDD and CAT options

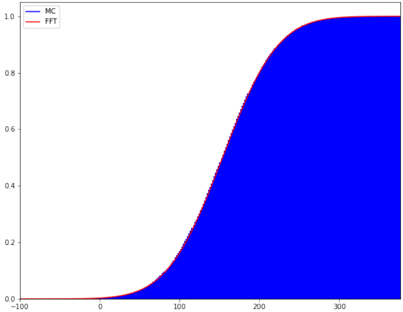


Figure: Cumulative distribution function of CAT for January 2019 and 30 days observation in advance computed by the FFT method (red) and Monte-Carlo with 50, 000 simulations (blue) $N = 2^{15}$.